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MODEL OF THE HELIOSPHERIC CURRENT SHEET OF FINITE THICKNESS

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Abstract
The unsteady-state model of the heliospheric current sheet of the finite thickness is suggested. The model generally includes all three components of the magnetic field. A difference of the suggested model from the standard one consists in the presence of the component of the magnetic field normal to the current sheet. The normal to the current sheet component of the magnetic field is connected with the unsteady radial component on the source surface of the heliospheric magnetic field. The effective thickness is defined by change of the tilt angle of the current sheet on the source surface during the solar rotation. The estimation of parameters of the model is based on calculations of the magnetic field on source surface within the potential approach (the model of Stanford University, USA). The model can be used in the theoretical studies of the galactic cosmic ray modulation in the heliosphere.

I. Introduction
The integral part of the modern model of the heliosphere is the heliospheric current sheet (HCS) - a surface on which there is a change of the direction of the heliospheric magnetic field (HMF). The HMF is usually described in the framework of Parker structure, which in the heliocentric inertial (nonrotational) spherical system of coordinates (K) includes a radial and azimuthal components B_r, B_φ, connected by the relation:

$$B_\varphi(\vec{r},t) = -B_r(\vec{r},t)\,\frac{\omega r \sin\theta}{V_r}, \qquad (1)$$

where ω is the angular velocity of the Sun, V_r is the velocity of the solar wind.

The radial magnetic field component is set at some distance r_0 from the Sun – on the source surface (SS). Thus, if $\Phi(\vec{r},t) = 0$ is the equation of the HCS, the traditional form of the HMF description everywhere in the heliosphere is given by expression:

$$\vec{B} = \vec{B}^m(\vec{r},t)\,[1 - 2H(\Phi(\vec{r},t))], \qquad (2)$$

where $\vec{B}^m(\vec{r},t)$ is the two-component monopol Parker HMF, directed everywhere out of the Sun and H is the Heaviside function. The equation (2) describes the HMF with the infinitely thin HCS (the mathematical surface). Such form of the description reflects the most essential properties of the "real" HCS defined according to measurements - its thickness is the least physical scale in the heliosphere for the galactic cosmic ray (GCR) particles. However, due to changes in form of the HCS in the consecutive solar rotations, the effective geometrical and physical characteristics of the HMF may differ significantly from those described by (2).

For the greater part of the 11-year solar activity cycle the HCS surface in the heliosphere is rather complicated. Often therefore it is approximated as a surface resulting from the reflection to the heliosphere of the great circle on SS, inclined by some tilt $\alpha(t) = (\lambda_{\max} - \lambda_{\min})/2$ to the equator. Here $\lambda_{\max}$, $\lambda_{\min}$ is the maximum and minimum HCS latitudes on the SS. Coordinates of the large circle are linked by the simple relation $\theta = \pi/2 - arctg\,(tg\alpha \sin\varphi)$. Then the HCS surface at any distance r can be described as $\theta = \pi/2 - arctg\,(tg\alpha \sin\tilde{\varphi})$, where $\tilde{\varphi} = \varphi - \omega t + \omega(r - r_0)/V_r$. This HCS model is usually called tilted current sheet (TCS) model. It is widely used for the HMF description during the periods of low solar activity, when the tilt is small ($\alpha(t)$<30°). The specification of HMF in the approximation of the TCS model corresponds to the dipole approximation.

The HCS in the TCS model is considered within several years near solar activity minimum as a steady structure with unique changing parameter – the tilt angle of the current sheet. The question of its stability is natural. From the point of view of the MHD theory [1] for the steady HCS the normal to it HMF component is necessary, as it provides the electric field maintaining the current, $\vec{E} = -\vec{V}_{sw} \times \vec{B}/c$. From the more general considerations [2-5] the presence of the normal to HCS component magnetic field is physically more natural, than its absence.

In [6] we estimated the normal component of the magnetic field that can be expected from the observed movement of HCS on the SS. Another physical reason for the occurrence of the normal to the HCS of the heliospheric magnetic field components, which can be investigated within the limits of simple MHD approach is the unsteady tilt angle of the sheet. The tilt of the sheet varies in the solar activity cycle in rather wide limits ($0^o \le \alpha(t) \le 75^o$). The change of the tilt results in unsteady radial HMF component on SS, which, in turn, results in the induction of the normal component of the HMF. This case is considered below.

II. The magnetic field on the source surface

The HMF is defined as the decision of the MHD equations in the assumption of its full freezing in the plasma of a solar wind [2,3]:

$$\vec{\nabla}\vec{B} = 0; \quad \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B}). \qquad (3)$$

In the specific case $\vec{V} = V_r \vec{e}_r$, $V_r = const$ the decision of (3) is well-known [3] and may be written as:

$$B_r(r,\theta,\varphi,t) = (r_0^2 / r^2)\, B_r(r_0,\theta,\tilde{\varphi},t-(r-r_0)/V_r);$$
$$B_\theta(r,\theta,\varphi,t) = (r_0 / r)\, B_\theta(r_0,\theta,\tilde{\varphi},t-(r-r_0)/V_r);$$
$$B_\varphi(r,\theta,\varphi,t) = (r_0 / r)\, B_\varphi(r_0,\theta,\tilde{\varphi},t-(r-r_0)/V_r). \qquad (4)$$

From (4) it follows that the components of the HMF in the heliosphere are determined by its values on the SS.

For the HMF analysis on SS it is convenient to use a combination of the equations (3), which may be written as in [3]:

$$\frac{\partial B_r}{\partial t} = \frac{V_r}{r_0 \sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, B_\theta) + \frac{\partial B_\varphi}{\partial\varphi}\right] \qquad (5)$$

The solution of the equation (5) is divergence-free. Time derivative of the radial HMF component is convenient to present as the sum:

$$\frac{\partial B_r}{\partial t} = \left(\frac{\partial B_r}{\partial t}\right)_c - \omega\frac{\partial B_r}{\partial\varphi}, \qquad (6)$$

where the first term on the right is connected with the unsteady-state in the system of coordinates rotating with the Sun (K^c), and the second term describes the contribution from rotation in the K system. Substituting (6) in (5) we receive the equation where all variables are related to K^c system:

$$\frac{\partial B_r}{\partial t} = \frac{V_r}{r_0 \sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, B_\theta) + \frac{\partial B_\varphi}{\partial\varphi}\right];$$
$$B_\varphi = \tilde{B}_\varphi + \frac{\omega r_0 \sin\theta}{V_r} B_r \qquad (7)$$

where $\tilde{B}_\varphi$ is an azimuthal component in motionless K system.

In the useful K' system with axis $OX' = -OZ^c$, $OY' = OY^c$, $OZ' = OX^c$ the first equation (7) becomes simpler:

$$\frac{\partial B_r}{\partial t} = \frac{V_r}{r_0 \sin\theta'}\frac{\partial B_{\varphi'}}{\partial\varphi'}. \qquad (8)$$

As the HCS on SS is a mathematical line, in the simple case of the homogeneous distribution of the radial HMF component within each hemisphere it may be written as:

$$B_r = B_0\{1 - 2H[\cos(\varphi' - \alpha(t) - \pi/2)]\}, \qquad (9)$$

where B_0 is a constant, $\alpha(t)$ is the instantaneous value of the tilt at the moment t. Substituting (9) in (8) results in

$$\frac{\partial B_{\varphi'}}{\partial\varphi'} = -\frac{2B_0 r_0 \sin\theta'}{V_r}\frac{d\alpha}{dt}\times \qquad (10)$$
$$\times\, sign(\sin(\varphi' - \alpha(t) - \pi/2)\,\delta(\varphi' - \alpha(t) - \pi/2),$$

where δ is the delta-function and φ' is measured from axis OX'. The decision of (10) is as follows:

$$B_{\varphi'} = \frac{2B_0 r_0 \sin\theta'}{V_r}\left(\frac{d\alpha}{dt}\right)_{\alpha(t) = \varphi' - \pi/2} \times f(\varphi',t), \qquad (11)$$

where

$$f(\varphi',t) = -H(\varphi' - \alpha(t) - \pi/2) + H(\varphi' - \alpha_2 - \pi/2)$$
$$+ H(\varphi' - \alpha(t) - 3\pi/2) - H(\varphi' - \alpha_2 - 3\pi/2).$$

The dependence of the $f(\varphi',t)$ on the angular variable for the case when the tilt angle of the HCS varies from $\alpha(0) = \alpha_1 = 12^o$ to $\alpha(T) = \alpha_2 = 48^o$ is presented in the top panel of Fig.1.

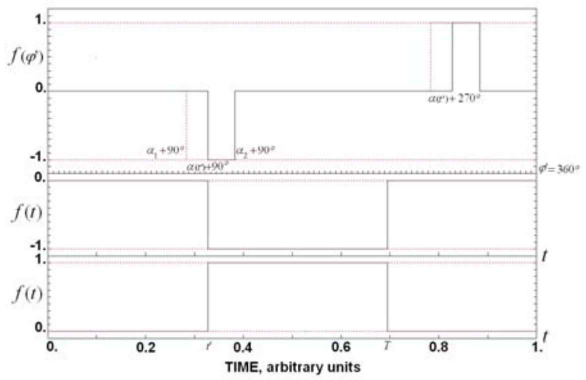

Fig.1.

The middle and lower panels show the dependence of the $f(\varphi',t)$ on time at φ' and $\varphi' + 180^o$, respectively.

The tilt angle of the HCS $\alpha(t)$ is defined on the time scale equal to the period of solar rotation T, therefore it is interesting to consider the time averages of (9) and (11) for the solar rotation. As a specific $\alpha(t)$ we use physically clear and simple model when $\alpha(t)$ increases with constant acceleration during the first half of solar rotation and then decreases with constant deceleration during the second half, i.e.

$$\frac{d\alpha}{dt} = 4\langle\dot{\alpha}\rangle\begin{cases}\tilde{t}, & 0 \le \tilde{t} \le 1/2\\ (1-\tilde{t}), & 1/2 < \tilde{t} \le 1\end{cases}, \qquad (12)$$

where $\langle\dot{\alpha}\rangle = \dfrac{(\alpha_2 - \alpha_1)}{T}$; $\quad \tilde{t} = t / T$.

Then the average components on the source surface can expressed as:

$$\langle B_r \rangle = \frac{1}{T}\int_0^T B_r \, dt = \mp B_0 (1 - 2\tilde{t}); \qquad (13)$$

$$\langle B_{\varphi'} \rangle = \frac{1}{T}\int_0^T B_{\varphi'} \, dt = \pm \frac{2B_0 r_0}{V_r}\left(\frac{d\alpha}{dt}\right)_{\alpha(t)+\pi/2=\varphi'} \cdot (1 - \tilde{t}),$$

where the signs correspond to $\varphi' = \alpha(t) + \pi/2$ and $\varphi' = \alpha(t) + 3\pi/2$, respectively, and $\tilde{t}$ is a relative time of passage of the current sheet through any angular coordinate φ'. It can be expressed as:

$$\tilde{t} = \begin{cases} (1/2)\sqrt{\Delta\varphi'/\langle\dot{\alpha}\rangle / T}\,, \; 0 \le \Delta\varphi' \le \langle\dot{\alpha}\rangle T & (14) \\ 1 - (1/2)\sqrt{2 - \Delta\varphi'/\langle\dot{\alpha}\rangle / T}\,, \; \langle\dot{\alpha}\rangle \cdot T \le \Delta\varphi' \le 2\langle\dot{\alpha}\rangle T \end{cases},$$

where $\Delta\varphi' = \varphi' - \pi/2 - \alpha_1$.

Because of (7) the averaged HMF components on the SS in the K system may be written as:

$$\langle B_r \rangle = B_0 (2\tilde{t} - 1);$$

$$\langle B_\theta \rangle = \frac{2B_0 r_0}{V_r}\left(\frac{d\alpha}{dt}\right)_{\tilde{t}} (1 - \tilde{t}) \cdot \sin\varphi \,;$$

$$\langle B_\varphi \rangle = -\frac{B_0\, r_0 \omega \sin\theta}{V_r}(2\tilde{t} - 1) + \frac{2B_0 r_0}{V_r}\left(\frac{d\alpha}{dt}\right)_{\tilde{t}} \times$$

$$\times (1 - \tilde{t}) \cdot \cos\theta \cdot \cos\varphi \,, \qquad (15)$$

where the coordinate φ is linked with the coordinate θ by the relation $\theta = \pi/2 - arctg\,(tg\alpha(\tilde{t}) \cdot \sin\varphi)$.

The HMF components in the heliosphere are determined according to (4). From expressions (15) it follows that for $d\alpha / dt > 0$ the field lines of the HMF within the HCS leave the SS and then come back. For $d\alpha / dt < 0$, on the contrary, the field lines come from the infinity to SS and then come back.

As the time averaging (13) switches with the divergence operation,

$$\vec{\nabla}\left(\frac{1}{T}\int_0^T \vec{B}\, dt\right) = \frac{1}{T}\int_0^T (\vec{\nabla}\vec{B})\, dt = 0\,, \qquad (16)$$

the average components of the HMF are divergence-free.

III. The parameters of the model and characteristic behavior of the HMF

The normal to the HCS components of the HMF are proportional to $\langle\dot{\alpha}\rangle / \omega$. This parameter can be estimated using the Wilcox Solar Observatory (WSO) model based on the measurements of the photospheric magnetic field and defining in the potential MHD approach the HCS tilt angle for each solar rotation (the site http://wso.stanford.edu. Tilt.html). In Fig. 2. the top panel presents the time behavior of the tilt angle α_t. The middle panel of Fig.2. shows the time dependence of the parameter α_d - the angle between of the magnetic dipole and the axis of rotation of the Sun. Note, that α_d is equal to α used in (9-15). In the bottom panel the time behavior $\langle\dot{\alpha}_d\rangle / \omega$ for the last three 11-year solar activity cycles is shown. It can be seen that the maximum value does not exceed 0.015 with the characteristic average value $\approx 3. \times 10^{-3}$. Thus, the ratio of the normal to the HCS components of the HMF to its radial component on SS is less than one percent, according to measurements.

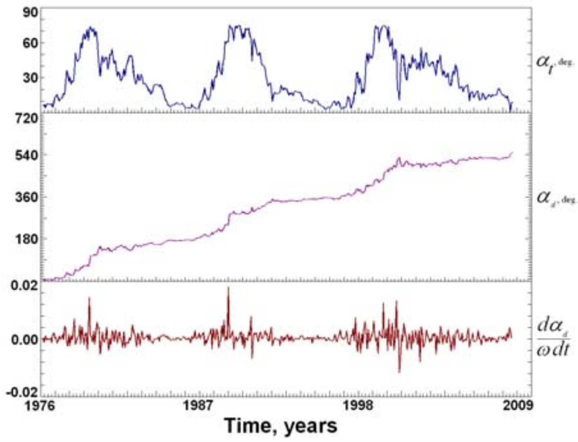

Fig.2.

For more detailed illustration of the angular distribution of the HMF components within the effective thickness of the HCS, we use $\langle\alpha\rangle = 30^o$ and much longer $\langle\dot{\alpha}\rangle / \omega = 0.1$. In Fig. 3. the latitudinal distributions of the HMF components on SS in relative units are presented. For B_φ the distribution of only the second term in (15) is shown. The azimuth angle is taken close to $\pi/2$, where the B_θ - component is maximal and B_φ is very small.

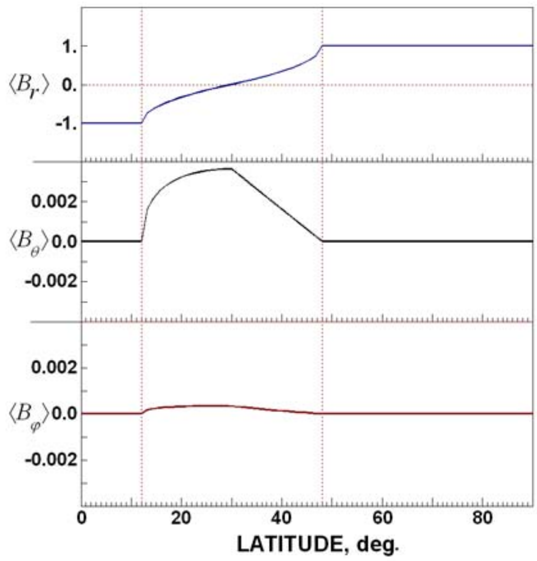

Fig.3.

Fig. 3. clearly shows that the ratio of the normal component to the tangential one strongly varies within the thickness of the HCS. At $\alpha(t) = \langle\alpha\rangle$ there is only the normal component. The ratio of the maximum value of the normal component to the maximum value of the tangential component is (3-4)$\times 10^{-3}$, as mentioned earlier on the basis of the measurement data.

Consistently with (15) on SS each component of HMF has a characteristic angular dependence within the HCS. Fig.4. presents the angular distribution of all three components of HMF on the SS without azimuthal component due to solar rotation.

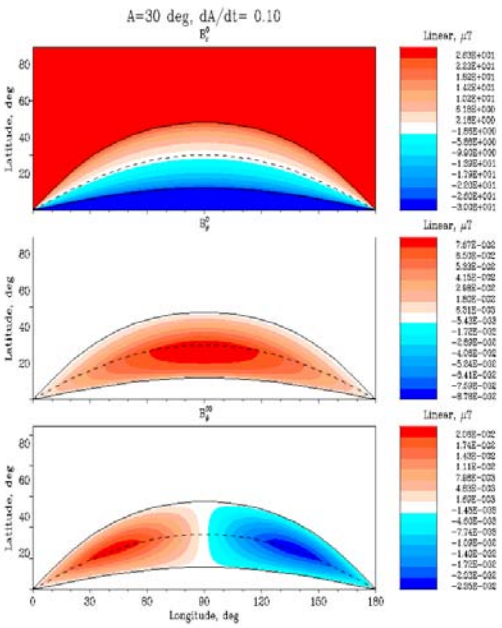

Fig. 4.

IV. Discussion

In spite of relatively small normal to the HCS component, its presence can lead to considerable effects in the transportation of the GCR particles from one magnetic hemisphere to another. Due to its slower reduction with the radial distance (~1/r) when compared with that of the radial component (~1/r^2), the role of the normal component of HMF significantly increases in the outer heliosphere [6]. It can lead the essential changes in the GCR modulation, as the effects of drift along of the HCS are sensitive to the normal component of the HMF.

The analysis of the "real" forms of the HCS on SS according to the WSO data shows, that the modelling description of the HCS form by means of the tilt angle does not fully reflect the evolution of the "real" form of the HCS in time [7]. The change from rotation to rotation of the mean HCS polar angle, according to [7], greatly exceeds the change in tilt angle of the HCS. Consequently the normal component of the HMF increases. Therefore, the description of change in the HCS by the change of its form from one rotation to the next one is more promising.

V. Conclusions

1. Based on the traditional model of tilted current sheet the analitical heliospheric current sheet model with the finite thickness and all three components of the magnetic field is constructed. The procedure of averaging the instantaneous magnetic field over the time to produce the average heliospheric current sheet for the solar rotation is introduced.

2. When using the parameters obtained from the Wilcox Solar Observatory data changes in the heliospheric magnetic field according to the present model are very small near the Sun, but should be greater at large heliospheric distances.

3. The alternative model of the heliospheric current sheet with the finite thickness based on the change of its form from one solar rotation to the next one can be more promising for the studies of the modulation of the galactic cosmic rays.

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